Inverse Filtering of Room Acoustics

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Abstract—**A novel method is proposed for realizing exact inverse filtering of acoustic impulse responses in a room. This method is based on the principle called the *m*ultiple-input/output *in*verse *t*heorem (MINT). Because a room impulse response generally has nonminimum phases, it has been impossible to realize exact inverse filtering of room acoustics using previously reported methods. However, the exact inverse of room acoustics can be realized using the proposed method. With this method, the inverse is constructed from multiple FIR filters (transversal filters) by adding some extra acoustic signal-transmission channels produced by multiple loudspeakers or microphones. The coefficients of these FIR filters can be computed by the well-known rules of matrix algebra. Inverse filtering in a sound field is investigated experimentally. It is shown that the proposed method is greatly superior to previous methods that use only one acoustic signal-transmission channel. The results in this paper prove the possibility of sound reproduction and sound receiving without any distortion caused by reflected sounds in a room.**

I. Introduction

GENERALLY, acoustic signals radiated inside a room are linearly distorted by wall reflections. These distortions, which arise as results of reverberations and echos, often spoil speech intelligibility. In addition, they are also undesirable when reproducing a desired sound field in a room. The way of removing these distortions is to realize the inverse of a room impulse response. Therefore, there is an unfulfilled need for an inverse-filtering method intended for room acoustics.

Consider the acoustic system consisting of loudspeaker S_1 and microphone M, as shown in Fig. 1. The transfer function of the acoustic signal-transmission channel between S_1 and M is denoted as $G(z^{-1})$. $G(z^{-1})$ represents the reflective sounds as well as the direct sound between S_1 and M.

It would appear that the inverse of this system could be constructed from the inverse filter $H(z^{-1})$ that satisfies the following expression:

$$H(z^{-1}) = 1/G(z^{-1}). \qquad (1)$$

However, this inverse becomes unstable because the acoustic signal-transmission channel $G(z^{-1})$ is generally considered to be a nonminimum phase function [1].

A number of inverse-filtering methods [2]–[5] have been reported. However, they cannot realize the exact inverse of an acoustic system that has nonminimum phases.

Manuscript received December 1, 1986; revised July 28, 1987.
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IEEE Log Number 8718017.

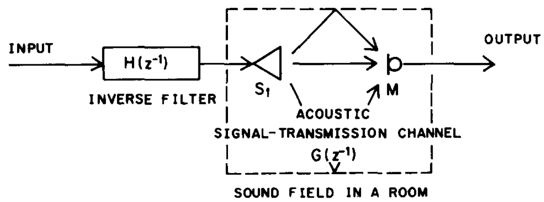

Fig. 1. Acoustic system consisting of a loudspeaker and a microphone. S_1: loudspeaker, M: microphone, $G(z^{-1})$: acoustic signal-transmission channel that corresponds to a room impulse response.

Most of them are based on the "least squares error (LSE)" criterion [2]–[4]. According to these conventional methods (LSE methods), the inverse of an acoustic system can be constructed from a stable FIR filter (transversal filter). However, this "inverse" is not the exact inverse but rather an approximate inverse of the system.

In this paper, a novel method is proposed for realizing exact inverse filtering of an acoustic system. In this method, an acoustic system is considered to be a multiple-input (or multiple-output) linear finite impulse response (FIR) system by using multiple loudspeakers (or microphones). This concept is not found in the conventional LSE method that uses only one acoustic signal-transmission channel.

The outline of this paper is as follows. Section II reviews the conventional LSE method for achieving inverse filtering of a nonminimum phase system. Section III describes the principle of the proposed method called the MINT. Section IV introduces a method for computing the inverse of a linear FIR system based on the MINT. Section V discusses an inverse-filtering experiment in a sound field. Also, the performance of the proposed method and the LSE method are compared.

II. Review of Conventional Inverse-Filtering Method

Consider the single-input single-output linear FIR system shown in Fig. 2. The impulse response of the system, $g(k)$, is assumed to have nonminimum phases, where k is a nonnegative integer index. The FIR filter is connected to the input of the system and its coefficients are denoted as $h(k)$.

When the filter is the inverse of the system, $g(k)$ and $h(k)$ must satisfy the relationship

$$d(k) = g(k) \circledast h(k), \qquad (2)$$

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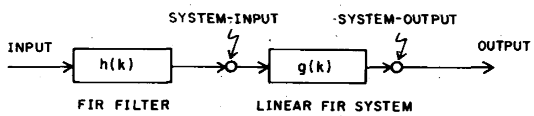

Fig. 2. Conventional inverse-filtering method based on the least squares error criterion (LSE method).

where

$$d(k) = \begin{cases} 1 & \text{when } k = 0 \\ 0 & \text{when } k = 1, 2, \cdots . \end{cases}$$

and $\circledast$ denotes the discrete linear convolution. Equation (2) can be expressed in matrix form as

$$\underbrace{\begin{bmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix}}_{L+1} = \underbrace{\begin{bmatrix} g(0) & & \\ g(1) & g(0) & 0 \\ \vdots & g(1) & \ddots \\ g(m) & \vdots & g(0) \\ & g(m) & g(1) \\ 0 & \ddots & \vdots \\ & & g(m) \end{bmatrix}}_{i+1} \begin{bmatrix} h(0) \\ h(1) \\ \vdots \\ \vdots \\ h(i) \end{bmatrix}, \tag{3a}$$

or

$$D = GH, \tag{3b}$$

where

$$L = i + m, \tag{4}$$

and

m: order of the z-transform of $g(k)$,
i: order of the FIR filter,
D: $(L + 1) \times 1$ column vector,
H: $(i + 1) \times 1$ column vector,

and

G: $(L + 1) \times (i + 1)$ matrix.

Here, there is no solution for (3) because the number of the columns is less than that of the rows in matrix G, as shown in the expression

$$L + 1 = m + i + 1 > i + 1. \tag{5}$$

In the conventional LSE method, the coefficients of the FIR filter are computed as the approximate solution of (3) by the relationship

$$H = (G^T G)^{-1} G^T D, \tag{6}$$

where G^T is the transposed matrix of G. Therefore, it is impossible to realize the exact inverse of a linear FIR system using this method.

In addition, no matter how high the order of the inverse filter might be, error energy $(D - GH)^T(D - GH)$ does not converge to 0, since $g(k)$ has nonminimum phases (see Appendix A). Accordingly, the "inverse-filter" obtained by the conventional LSE method cannot accurately approximate the "inverse" of a nonminimum phase system.

III. Principle of Proposed Inverse-Filtering Method

A. Fundamental Principle

The drawback in the conventional LSE method seems to result from the use of only one signal-transmission channel. However, many systems in room acoustics, some electric circuits, and so on can be modified to multiple-input linear FIR systems by adding some extra signal-transmission channels. In these cases, the exact inverse of the system can be constructed by applying the principle [6] introduced in this section.

Consider the two-input single-output linear FIR system shown in Fig. 3. This system can be obtained by adding an extra signal-transmission channel to the linear system shown in Fig. 2. The two signal-transmission channels of this system are denoted as $G_1(z^{-1})$ and $G_2(z^{-1})$, and the two FIR filters $H_1(z^{-1})$ and $H_2(z^{-1})$ are connected to the inputs of $G_1(z^{-1})$ and $G_2(z^{-1})$.

To realize inverse filtering of the system, $H_1(z^{-1})$ and $H_2(z^{-1})$ must satisfy the expression

$$D(z^{-1}) = 1 = G_1(z^{-1}) H_1(z^{-1}) + G_2(z^{-1}) H_2(z^{-1}), \tag{7}$$

where

$D(z^{-1})$: z-transform of $d(k)$ in (2).

Since $G_1(z^{-1})$, $G_2(z^{-1})$, $H_1(z^{-1})$, and $H_2(z^{-1})$ are polynomials of z^{-1}, a solution set for (7), $(H_1(z^{-1}), H_2(z^{-1}))$, has the following two properties.

a) Solutions for (7) exist when and only when $G_1(z^{-1})$ and $G_2(z^{-1})$ are relatively prime (in other words, $G_1(z^{-1})$ and $G_2(z^{-1})$ do not have any common zero in the z-plane).
b) When (7) has a solution, it is unique under the requirement that the orders of $H_1(z^{-1})$ and $H_2(z^{-1})$ are less than those of $G_2(z^{-1})$ and $G_1(z^{-1})$, respectively.

Therefore, there exists a pair of FIR filters, $H_1(z^{-1})$ and $H_2(z^{-1})$, that can realize exact inverse filtering of a two-input single-output linear FIR system (The proof of the properties is shown in Appendix B.)

This principle can be applied to sound reproduction in a sound field in a room. Consider the acoustic system shown in Fig. 4. In this figure, $G_1(z^{-1})$ and $G_2(z^{-1})$ represent the acoustic signal-transmission channels between loudspeakers S_1, S_2 and receiving point M. The acoustic signals radiated from S_1 and S_2 are superposed at M after passing through $G_1(z^{-1})$ and $G_2(z^{-1})$.

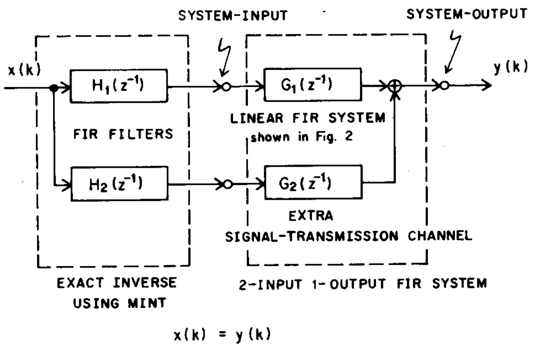

Fig. 3. Proposed inverse-filtering method based on the *m*ultiple-input/output *in*verse *t*heorem (MINT).

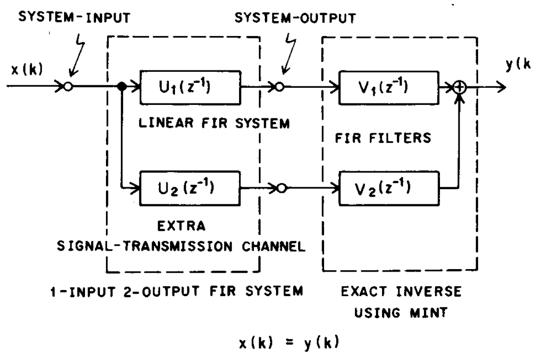

Fig. 5. Proposed inverse-filtering method applied to a single-input two-output system.

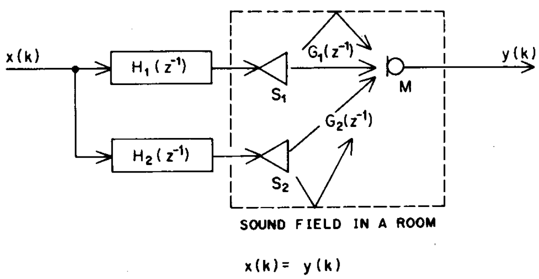

Fig. 4. Sound reproduction using the proposed method. S_1, S_2: loudspeaker, M: sound receiving point, $G_1(z^{-1})$, $G_2(z^{-1})$: acoustic signal-transmission channel.

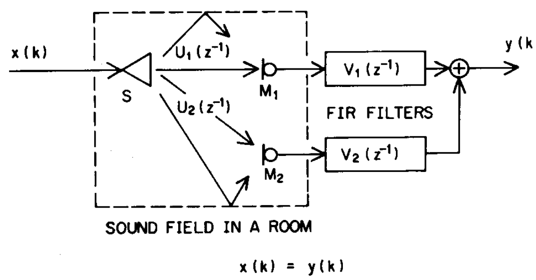

Fig. 6. Dereverberation using the proposed method. S: sound-source, M_1, M_2: microphone, $U_1(z^{-1})$, $U_2(z^{-1})$: acoustic signal-transmission channel.

When $G_1(z^{-1})$ and $G_2(z^{-1})$ do not have any common zero, there exist FIR filters $H_1(z^{-1})$ and $H_2(z^{-1})$ that satisfy the relationship in (7). Hence, exact inverse filtering is realized by connecting $H_1(z^{-1})$ and $H_2(z^{-1})$ to the inputs of S_1 and S_2, respectively. Therefore, it becomes possible to reproduce the desired acoustic signals at M without any distortion caused by wall reflections using the proposed principle.

It is also possible to apply the principle to a single-input two-output linear FIR system for reconstructing the input signals of the system. A block diagram of the system is illustrated in Fig. 5. The system's two signal-transmission channels are denoted as $U_1(z^{-1})$ and $U_2(z^{-1})$. Two FIR filters $V_1(z^{-1})$ and $V_2(z^{-1})$ are assumed to be connected to the outputs of $U_1(z^{-1})$ and $U_2(z^{-1})$, respectively.

To reconstruct the input signal of the system, $V_1(z^{-1})$ and $V_2(z^{-1})$ must satisfy the expression

$$1 = U_1(z^{-1})\,V_1(z^{-1}) + U_2(z^{-1})\,V_2(z^{-1}). \quad (8)$$

This equation is identical with (7). Hence, the same principle mentioned above can be applied to prove the existence of FIR filters $V_1(z^{-1})$ and $V_2(z^{-1})$. Accordingly, the principle is applicable to reconstruction of the input signal of a single-input two-output linear FIR system from its output signals.

In room acoustics, this concept is useful for a microphone system to dereverberate the acoustic signals received inside a room. Consider the acoustic system shown in Fig. 6. $U_1(z^{-1})$ and $U_2(z^{-1})$ denote the acoustic signal-transmission channels from sound source S to microphones M_1 and M_2, respectively. The acoustic signals radiated from S are received by microphones M_1 and M_2. Then, output signals from M_1 and M_2 are summed in the adder.

This system can be considered to be equivalent to the single-input two-output linear FIR system mentioned before. Hence, the output signals of the adder and the direct sound from S become the same by using FIR filters $V_1(z^{-1})$ and $V_2(z^{-1})$ that satisfy (8). Therefore, there is a strong possibility that the proposed principle can be applied to a method for dereverberation on the acoustic signals received by multiple microphones inside a room.

B. Extension of the Principle

Here, the above-mentioned principle is extended for inverting a multiple-input multiple-output linear FIR system. In order to create the desired sound field, a concept seems to be necessary to cancel the effects of room impulse responses at multiple points in a room. This concept is also useful for realizing an effect similar to the "Cocktail Party Effect" with a microphone system. Hence, it appears important to extend the proposed principle.

Consider the $n+1$-input n-output ($n = 2, 3, \cdots$) system shown in Fig. 7. In this figure, $G_{ij}(z^{-1})$ ($i = 1, 2, \cdots, n+1$; $j = 1, 2, \cdots, n$) is denoted as a signal-transmission channel between the ith input and the jth output of the system. $H_{ij}(z^{-1})$ denotes an FIR filter connected to the ith input of the system.

Inverse filtering of the jth output of the system can be defined by the expression

$$\underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}}_{n} = \begin{bmatrix} G_{11}(z^{-1}) & \cdots & G_{n+11}(z^{-1}) \\ G_{12}(z^{-1}) & \cdots & G_{n+12}(z^{-1}) \\ \vdots & & \vdots \\ G_{1j}(z^{-1}) & \cdots & G_{n+1j}(z^{-1}) \\ \vdots & & \vdots \\ \vdots & & \vdots \\ G_{1n}(z^{-1}) & \cdots & G_{n+1n}(z^{-1}) \end{bmatrix}_{\leftarrow n+1 \rightarrow} \begin{bmatrix} H_{1j}(z^{-1}) \\ H_{2j}(z^{-1}) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ H_{nj}(z^{-1}) \\ H_{n+1j}(z^{-1}) \end{bmatrix} \tag{9a}$$

or

$$R_j = GH_j, \tag{9b}$$

where

R_j: $n \times 1$ column vector,
G: $n \times (n+1)$ matrix such as

$$G = [G_1 \quad G_2 \quad \cdots \quad G_{n+1}],$$

where G_i ($i = 1, 2, \cdots, n+1$) denotes the ith column vector in matrix G, and

H_j: $(n \times 1) \times 1$ column vector.

Equation (9) has the following meaning.

1) The jth output of the system can be inverted using FIR filters $H_{ij}(z^{-1})$ ($i = 1, 2, \cdots, n+1$) independently of the other outputs.

2) Solutions for (9) $H_{ij}(z^{-1})$ exist when the Smith canonical form [7] of G can be represented as matrix $[I_n \; 0]$, where I_n denotes the $n \times n$ unit matrix and 0 is an $n \times 1$ column vector with all zero elements (see Appendix C).

Accordingly, it is possible to realize the exact inverse of a multiple-input multiple-output linear FIR system by the proposed principle called MINT.

IV. Computation of FIR Filters for Exact Inverse

This section describes the computation of the FIR filters introduced in the previous section. To simplify the explanation, inverse filtering of the two-input single-output system shown in Fig. 3 is considered.

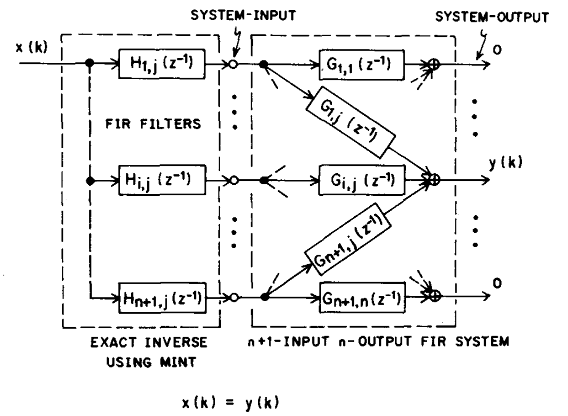

Fig. 7. Proposed inverse-filtering method for a multiple-input multiple-output system.

Equation (7) can be rewritten as

$$d(k) = g_1(k) \circledast h_1(k) + g_2(k) \circledast h_2(k), \tag{10}$$

where

$g_1(k), g_2(k)$: impulse responses of $G_1(z^{-1})$ and $G_2(z^{-1})$,
$h_1(k), h_2(k)$: coefficients of $H_1(z^{-1})$ and $H_2(z^{-1})$.

This equation can be expressed in matrix form as

$$D = G_1H_1 + G_2H_2 = [G_1 \quad G_2] \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \tag{11a}$$

or

$$\underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}}_{L+1} = \begin{bmatrix} g_1(0) & & & g_2(0) & & \\ g_1(1) & \ddots & 0 & g_2(1) & \ddots & 0 \\ \vdots & & & \vdots & & \\ \vdots & & g_1(0) & g_2(n) & & \\ g_1(m) & \ddots & g_1(1) & & & g_2(0) \\ & & \vdots & & & g_2(1) \\ 0 & & \vdots & 0 & \ddots & \vdots \\ & & g_1(m) & & & g_2(n) \end{bmatrix}_{\leftarrow i+1 \rightarrow \; \leftarrow j+1 \rightarrow} \begin{bmatrix} h_1(0) \\ h_1(1) \\ \vdots \\ \vdots \\ h_1(i) \\ h_2(0) \\ \vdots \\ h_2(j) \end{bmatrix}, \tag{11b}$$

where

$$L = m + i = n + j, \quad (12)$$

and

$m+1, n+1$: durations of $g_1(k)$ and $g_2(k)$,
i, j: orders of $H_1(z^{-1})$ and $H_2(z^{-1})$,
D: $(L+1) \times 1$ column vector,
$[H_1^T \;\; H_2^T]^T$: $(i+j+2) \times 1$ column vector,

and

$[G_1 \;\; G_2]$: $(L+1) \times (i+j+2)$ matrix.

Here, $[G_1 \; G_2]$ becomes a square matrix when orders i and j of two FIR filters are chosen to satisfy the equalities

$$i = n - 1$$

and

$$j = m - 1. \quad (13)$$

$[G_1 \; G_2]$ is a regular matrix because there exists a unique set of FIR filters that satisfies (10) and property b) described in Section III-A. Accordingly, the coefficients of the FIR filters, $h_1(k)$, $h_2(k)$, can be computed by the relationship

$$\begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = [G_1 \quad G_2]^{-1} D. \quad (14)$$

V. Inverse-Filtering Experiment in a Sound Field

In room acoustics, it has always been believed that there could be no method for removing the distortions caused by wall reflections. This is because a room impulse response is generally considered to have nonminimum phases. However, a sound field can be considered to be a multiple-input (or multiple-output) linear FIR system by employing multiple loudspeakers (or microphones). Hence, the proposed inverse-filtering method can be applied to that problem, when two room impulse responses do not have any common zero.

According to computer simulations using a sound field in a rectangular enclosure [8], the proposed method performed inverse filtering with sufficient accuracy by avoiding some symmetrical positions of sound sources and receiving points. Although more studies are necessary to foresee the possibility of a common zero between two room impulse responses, we believe that the method can be applied to most loudspeaker-microphone positions in any room except some symmetrical positions.

To verify the applicability of the proposed method to a sound field, an inverse-filtering experiment was conducted in the frequency band 315–3150 Hz. The conventional LSE method was also applied to this problem in order to highlight the difference between the proposed inverse-filtering method and the LSE method.

The experimental conditions are shown in Fig. 8. Two reflectors were placed in an L-shaped arrangement in an anechoic room. Another reflector was also placed on the floor. Microphone M was placed 1 m from loudspeakers S_1 and S_2. The output of M was fed through a band-pass filter (BPF), intended for the frequency band 315–3150 Hz, to a digital computer that was used for computing the coefficients of the FIR filters $H_1(z^{-1})$ and $H_2(z^{-1})$. The acoustic signal transmission channel between S_1 and M (including S_1 and M) is denoted as $G_1(z^{-1})$, and that between S_2 and M is denoted as $G_2(z^{-1})$.

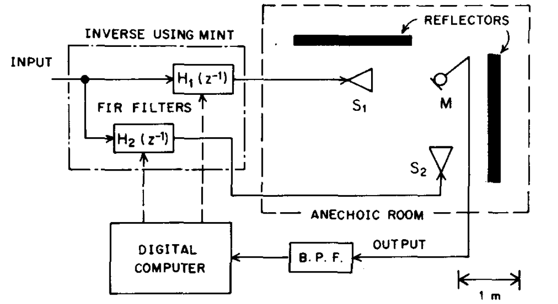

Fig. 8. Experimental conditions for inverse filtering in a sound field. S_1, S_2: loudspeaker, M: microphone, BPF: band-pass filter intended for 315–3150 Hz. The number of taps of FIR filter $H_i(z^{-1})$ ($i = 1$ or 2) in the LSE method is 700, and that of $H_i(z^{-1})$ in the proposed method is 350.

In this experiment, the desired impulse response $D(z^{-1})$ [see (7)] was arranged as the impulse response of the BPF. The errors, $E_m(z^{-1})$ and $E_i(z^{-1})$ ($i = 1$ or 2), between $D(z^{-1})$ and the impulse responses caused by the proposed inverse-filtering method and the LSE method were compared. $E_m(z^{-1})$ and $E_i(z^{-1})$ are represented by the expressions

$$E_m(z^{-1}) = D(z^{-1}) - \{G_1(z^{-1})H_1(z^{-1}) + G_2(z^{-1})H_2(z^{-1})\}, \quad (15a)$$

$$E_i(z^{-1}) = D(z^{-1}) - G_i(z^{-1})H_i(z^{-1}), \quad (15b)$$

where

$H_1(z^{-1})$ and $H_2(z^{-1})$: the FIR filters constructed by the proposed method,
$H_i(z^{-1})$ $(i = 1, 2)$: the FIR filter constructed by the LSE method when signal-transmission channel $G_i(z^{-1})$ is considered.

The results of this experiment are shown in Fig. 9. Here, curves (a), (b), (c), and (d) show the power spectra of $D(z^{-1})$, $E_1(z^{-1})$, $E_2(z^{-1})$, and $E_m(z^{-1})$, respectively. From these results, the following conclusions can be drawn.

- The difference between curves (a) and (b) shows that the performance of the LSE method is deeply influenced by the nature of the acoustic signal-transmission channel in use.
- The error using the proposed method is about 40 dB less than the error obtained by the LSE method at almost all frequencies.

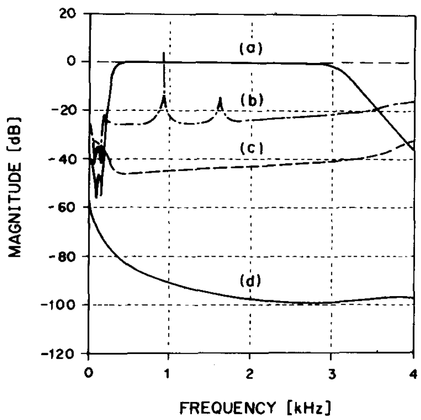

Fig. 9. Power spectra of errors in the LSE method and the proposed inverse-filtering method under the conditions shown in Fig. 8: (a) desired response, (b) and (c) error in the LSE method, (d) error in the proposed method [see (15)].

The error in the proposed method is thought to be due to the accuracy limits of the digital computer used in this experiment. Even with this error, the proposed method has been shown to be vastly superior to the LSE method to inverse filtering of room acoustics.

VI. Conclusion

An inverse-filtering method has been proposed for achieving the exact inverse of room acoustics that have nonminimum phases. This method is based on the principle called MINT. According to this principle, the inverse is realized using multiple FIR filters and multiple loudspeakers (or microphones). Coefficients of the filters can easily be determined by the well-known rules of matrix algebra as described in Section IV. The inverse realized by the proposed method is useful in removing any distortion due to wall reflections.

An experiment was conducted for realizing inverse filtering at a point in a sound field. This experiment clarified that the proposed method is vastly superior to a conventional method that uses only one acoustic signal-transmission channel.

We expect that the method will become a powerful one for producing the desired sound field in a room as well as dereverberating the acoustic signals radiated inside a room.

Appendix A

Consider the inverse filter of a minimum phase function such as

$$g(z^{-1}) = g_0 + g_1 z^{-1} + \cdots + g_{n-1} z^{-(n-1)}. \tag{A1}$$

Since $g(z^{-1})$ is a minimum phase, the following relationship is satisfied:

$$g_0^2 > g_{n-1}^2. \tag{A2}$$

When the inverse filter of $g(z^{-1})$ is computed with the LSE method [see (6)], the error energy e_k^2 caused by it can be represented as

$$\begin{aligned} e_k^2 &= D_k^T D_k - D_k^T G_k (G_k^T G_k)^{-1} G_k^T D_k \\ &= 1 - g_0^2 D_k^T (G_k^T G_k)^{-1} D_k, \end{aligned} \tag{A3}$$

where $(k-1)$ denotes the order of the inverse filter and D_k denotes $k \times 1$ column vector $[1\ 0 \cdots 0]^T$. G_k is $(k + n - 1) \times k$ matrix composed of $g_0, g_1, \cdots, g_{n-1}$ and 0 like matrix G in (3). If k is greater than n, it is possible to rewrite this equation to the expression

$$e_k^2 = 1 - g_0^2 \cdot \det(G_{k-1}^T G_{k-1}) / \det(G_k^T G_k). \tag{A4}$$

The inverse of a minimum phase function can be constructed from an infinite impulse response (IIR) filter. Hence, the impulse response of the inverse filter of $g(z^{-1})$ computed with the LSE method converges to the impulse response of the IIR inverse of $g(z^{-1})$ as k increases in number. Therefore, the following relationship holds well:

$$\begin{aligned} \lim_{k\to\infty} \{e_k^2\} &= \lim_{k\to\infty} \{1 - g_0^2 \cdot \det(G_{k-1}^T G_{k-1}) / \det(G_k^T G_k)\} \\ &= 0. \end{aligned} \tag{A5}$$

$$\therefore \lim_{k\to\infty} \{\det(G_{k-1}^T G_{k-1}) / \det(G_k^T G_k)\} = 1/g_0^2. \tag{A6}$$

Next, consider the inverse of the maximum phase function,

$$\hat{g}(z^{-1}) = g_{n-1} + g_{n-2} z^{-1} + \cdots + g_0 z^{-(n-1)}. \tag{A7}$$

This function can be formed by reflecting outside the unit circle all zeros of $g(z^{-1})$ that are inside the unit circle in the z-plane, keeping the magnitude of $g(z^{-1})$ the same [9].

The error energy caused by the "inverse" of $\hat{g}(z^{-1})$ can be represented as

$$\begin{aligned} \hat{e}_k^2 &= 1 - g_{n-1}^2 D_k^T (\hat{G}_k^T \hat{G}_k)^{-1} D_k \\ &= 1 - g_{n-1}^2 \cdot \det(\hat{G}_{k-1}^T \hat{G}_{k-1}) / \det(\hat{G}_k^T \hat{G}_k), \end{aligned} \tag{A8}$$

where

$$\hat{G}_k = \underbrace{\begin{bmatrix} g_{n-1} & & \\ g_{n-2} & g_{n-1} & 0 \\ \vdots & g_{n-2} & \ddots \\ g_0 & \vdots & g_{n-1} \\ & g_0 & g_{n-2} \\ 0 & \ddots & \vdots \\ & & g_0 \end{bmatrix}}_{k}.$$

It is clear that det $(\hat{G}_{k-1}^T \hat{G}_{k-1})/\det(\hat{G}_k^T \hat{G}_k)$ is identical to det $(G_{k-1}^T G_{k-1})/\det(G_k^T G_k)$ in (A4). Therefore, the following relationship can be shown by considering (A2):

$$\lim_{k\to\infty} \{\hat{e}_k^2\} = \lim_{k\to\infty} \{1 - g_{n-1}^2 \cdot \det(G_{k-1}^T G_{k-1})/\det(G_k^T G_k)\} = 1 - g_{n-1}^2/g_0^2 \neq 0. \quad \text{(A9)}$$

A nonminimum phase function can be represented as the product of a minimum phase function by a maximum phase function. As mentioned above, the error energy caused by the "inverse" of the maximum phase function does not converge to 0. Therefore, in the LSE method no matter how high the order of the inverse filter might be, the error energy does not converge to 0 when a nonminimum phase function is considered.

Appendix B

According to the characteristics of the Euclidean algorithm [10], it can be ascertained that there exists a general solution set for (7) $(\hat{H}_1(z^{-1}), \hat{H}_2(z^{-1}))$ that satisfies the expressions

$$\hat{H}_1(z^{-1}) = H_1(z^{-1}) + G_2(z^{-1})\,K(z^{-1}),$$

and

$$\hat{H}_2(z^{-1}) = H_2(z^{-1}) - G_1(z^{-1})\,K(z^{-1}), \quad \text{(A10)}$$

and

$$\deg H_1(z^{-1}) < \deg G_2(z^{-1}),$$

and

$$\deg H_2(z^{-1}) < \deg G_1(z^{-1}), \quad \text{(A11)}$$

where $(H_1(z^{-1}), H_2(z^{-1}))$ is a solution set of (7) and $K(z^{-1})$ is an arbitrary polynomial of z^{-1}.

Suppose that there is another solution set of (7) $(\dot{H}_1(z^{-1}), \dot{H}_2(z^{-1}))$ that satisfies the relationship in (A11). Because of (A10), $\dot{H}_1(z^{-1})$ and $\dot{H}_2(z^{-1})$ must be expressed as

$$\dot{H}_1(z^{-1}) = H_1(z^{-1}) + G_2(z^{-1})\,K(z^{-1})$$

and

$$\dot{H}_2(z^{-1}) = H_2(z^{-1}) - G_1(z^{-1})\,K(z^{-1}). \quad \text{(A12)}$$

In this case,

$$\deg \dot{H}_1(z^{-1}) = \deg G_2(z^{-1})\,K(z^{-1}) > \deg G_2(z^{-1})$$

and

$$\deg \dot{H}_2(z^{-1}) = \deg G_1(z^{-1})\,K(z^{-1}) > \deg G_1(z^{-1}). \quad \text{(A13)}$$

This, however, contradicts the assumption. Therefore, there exists only one solution for (7) that satisfies relationship (A11).

Appendix C

$$R_j = [G_1 \quad G_2 \quad \cdots \quad G_n] \begin{bmatrix} H_{1j}(z^{-1}) \\ \cdots \\ H_{nj}(z^{-1}) \end{bmatrix} + G_{n+1}H_{n+1j}(z^{-1}), \quad \text{(A14a)}$$

or

$$R_j = G_n H_{nj} + G_{n+1}H_{n+1j}(z^{-1}), \quad \text{(A14b)}$$

where

G_n: $n \times n$ polynomial matrix,
H_{nj}: $n \times 1$ column vector,
$[G_n \quad G_{n+1}]$: matrix G in (9).

When the Smith canonical form [7] of $[G_n\, G_{n+1}]$ is equivalent to $n \times (n+1)$ matrix $[I_n\, 0]$ (in other words, when G_n and G_{n+1} are relatively left prime), there exist $n \times n$ matrix W_n and $1 \times n$ vector W_{n+1} that satisfy the relationship [7]

$$I_n = G_n W_n + G_{n+1} W_{n+1}. \quad \text{(A15)}$$

The following expression is given with respect to the jth column vector of I_n:

$$R_j = G_n W_j + G_{n+1} W_{n+1}, \quad \text{(A16)}$$

where W_j is the jth column vector in W_n. It is clear that this equation is identical to (A14). Therefore, FIR filters that can realize the exact inverse of a multiple-input multiple-output system exist when the Smith canonical form of G is equivalent to $[I_n\, 0]$.

Acknowledgment

The authors are greatly indebted for the technical advice and invaluable guidance of Dr. J. Ohga, Fujitsu Company Ltd., a former Research Staff member of NTT. They wish to express their gratitude to K. Kakehi, Head of the Audio Visual Perception and Cognition Group, Information Science Department, NTT Basic Research Laboratories, for his thoughtful comments and interest in this work. The continuous encouragement of Dr. S. Furui, Head of the 4th Section, Information Science Department, NTT Basic Research Laboratories, is also gratefully acknowledged.

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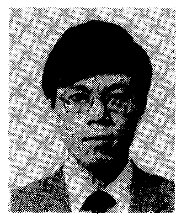

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